

# Radiative leptogenesis in minimal seesaw models\*

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In the framework of seesaw models with only two heavy Majorana neutrinos, nonzero leptonic asymmetries can be radiatively generated when exact heavy neutrino mass degeneracy ( $M_1 = M_2 = M$ ) is imposed at a scale  $\Lambda_D > M$ . For a specific case, we show that an acceptable value for the baryon asymmetry of the Universe can be obtained considering thermal leptogenesis.

Recently, a lot of attention has been paid to the study of a possible connection between thermal leptogenesis and neutrino masses [2], mixing and leptonic  $CP$  violation [3]. For instance, it is now known that an acceptable value for the baryon asymmetry of the Universe (BAU) requires the low-energy Majorana neutrinos to be lighter than  $0.12 - 0.15$  eV. Moreover, successful thermal leptogenesis implies  $M_1 > 10^8 - 10^9$  GeV [2,4], where  $M_1$  denotes the mass of the lightest heavy Majorana neutrino. Although interesting, these bounds are model-dependent in the sense that they are only valid if the heavy Majorana neutrino masses are hierarchical.

In supersymmetric theories, the above constraint on  $M_1$  may be in conflict with the upper bound on the reheating temperature of the Universe, which can be as low as  $10^6$  GeV [5]. This tension between the bounds on  $M_1$  and  $T_{RH}$  can be relaxed if one considers quasi-degenerate heavy Majorana neutrinos. In this case, acceptable values for the BAU can be obtained with heavy masses as low as 1 TeV [6].

In Ref. [1] we have studied the case where the small heavy Majorana neutrino mass-splitting, needed to enhance the  $CP$ -asymmetries, is generated radiatively. For simplicity, we have restricted ourselves to a seesaw model with only two

heavy right-handed neutrinos [7]. We denote the Dirac neutrino and charged-lepton Yukawa coupling matrices respectively by  $Y$  and  $Y_\ell$ , while  $M_R$  will stand for the  $2 \times 2$  symmetric mass matrix of the heavy right-handed neutrinos.

We start by considering that at a scale  $\Lambda_D$  the heavy neutrinos are degenerate, i.e.  $M_1 = M_2 \equiv M$ , with  $M < \Lambda_D$ . In this limit,  $CP$  is not necessarily conserved. Indeed, the non-vanishing of the weak-basis invariant  $\mathcal{J}_1 = M^{-6} \text{Tr} [Y Y^T Y_\ell Y_\ell^\dagger Y^* Y^\dagger, Y_\ell^* Y_\ell^T]^3$ , which is not proportional to  $M_2^2 - M_1^2$ , would signal a violation of  $CP$ . On the other hand, a non-zero leptonic asymmetry can be generated if and only if the  $CP$ -odd invariant  $\mathcal{J}_2 = \text{Im Tr} [H M_R^\dagger M_R M_R^\dagger H^T M_R]$  does not vanish [6]. Since  $\mathcal{J}_2$  can be expressed in the form

$$\mathcal{J}_2 = M_1 M_2 (M_2^2 - M_1^2) \text{Im} [H_{12}^2], \quad H = Y^\dagger Y, \quad (1)$$

$\mathcal{J}_2 \neq 0$  requires not only  $M_1 \neq M_2$  but also  $\text{Im} [H_{12}^2] \neq 0$ , at the leptogenesis scale  $M$ . Although the first condition is easily guaranteed by the running of  $M_R$  from  $\Lambda_D$  to  $M$ , the second one requires the inclusion of quantum corrections to the Dirac neutrino Yukawa matrix  $Y$ . The renormalization group equation (RGE) for  $M_R$  is, for the extended standard (SM) and minimal supersymmetric standard (MSSM) models [8],

$$\frac{dM_R}{dt} = c(H^T M_R + M_R H), \quad t = \frac{\ln(\mu/\Lambda_D)}{16\pi^2}, \quad (2)$$

with  $c_{SM} = 1$  and  $c_{MSSM} = 2$ .

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In the basis where  $M_R$  is diagonal, and assuming that the charged-lepton Yukawa matrix  $Y_\ell$  is diagonal, the evolution of  $Y$  and the heavy Majorana masses  $M_i$  is given at one-loop by

$$\frac{dM_i}{dt} = 2c M_i H_{ii}, \quad (3)$$

$$\frac{dY}{dt} = kY + \left[ -a Y_\ell Y_\ell^\dagger - b Y Y^\dagger \right] Y + Y T, \quad (4)$$

$$\frac{dH}{dt} = 2kH - 2bH^2 - 2aY^\dagger Y_\ell Y_\ell^\dagger Y + [H, T], \quad (5)$$

where  $k$  is a function of  $\text{Tr}(Y_X Y_X^\dagger)$  and the gauge couplings [9] and  $[H, T] = HT - TH$ . For the SM and MSSM, the factors  $a$  and  $b$  are

$$a_{\text{SM}} = -b_{\text{SM}} = \frac{3}{2}, \quad b_{\text{MSSM}} = 3a_{\text{MSSM}} = -3. \quad (6)$$

The matrix  $T$  is anti-Hermitian with  $T_{ii} = 0$  and

$$T_{12} = \frac{2 + \delta_N}{\delta_N} \text{Re}[H_{12}] + i \frac{\delta_N}{2 + \delta_N} \text{Im}[H_{12}]. \quad (7)$$

Here, the parameter  $\delta_N \equiv M_2/M_1 - 1$  quantifies the degree of degeneracy between  $M_1$  and  $M_2$ .

From Eq. (7) one can see that if  $\delta_N = 0$  at a given scale  $\Lambda_D$ , then the RGE in Eqs. (4) and (5) become singular, unless one imposes  $\text{Re}(H_{12}) = 0$ . This can be achieved by rotating the heavy fields by an orthogonal transformation  $O$ , being the rotation angle  $\theta$  such that

$$\tan 2\theta = 2 \text{Re}[H_{12}]/(H_{22} - H_{11}). \quad (8)$$

Under this transformation,  $Y \rightarrow Y' = YO$  and  $H \rightarrow H' = Y'^\dagger Y' = O^\dagger H O$ . It is straightforward to show that

$$H' = \begin{pmatrix} H_{11} - \Delta & i \text{Im}[H_{12}] \\ -i \text{Im}[H_{12}] & H_{22} + \Delta \end{pmatrix}, \quad (9)$$

where  $\Delta \equiv \tan \theta \text{Re}[H_{12}]$ . From  $\delta_N = M_2/M_1 - 1$  and Eq. (3) one has

$$\frac{d\delta_N}{dt} = 2c(\delta_N + 1)(H'_{22} - H'_{11}). \quad (10)$$

In the limit  $\delta_N \ll 1$ , the leading-log approximation for  $\delta_N(t)$  can be easily found to be

$$\delta_N(t) \simeq 2c(H'_{22} - H'_{11})t. \quad (11)$$

For quasi-degenerate Majorana neutrinos the  $CP$ -asymmetries generated in their decays are approximately given by [6]

$$\varepsilon_j = \frac{\text{Im}[H'_{21}]}{16\pi\delta_N H'_{jj}} \left( 1 + \frac{\Gamma_i^2}{4M^2\delta_N^2} \right)^{-1}, \quad j = 1, 2. \quad (12)$$

Here,  $\Gamma_i = H'_{ii}M_i/(8\pi)$  is the tree-level decay width of the heavy Majorana neutrino  $N_i$ . The above equation shows that

$$\varepsilon_i(t) \propto \text{Im}[H'_{12}(t)] \text{Re}[H'_{12}(t)], \quad i = 1, 2. \quad (13)$$

Therefore, a necessary condition to have a nonzero  $CP$ -asymmetry at a given  $t$  is that  $\text{Re}[H'_{12}(t)] \neq 0$ . Since  $\text{Re}[H'_{12}(0)] = 0$ , one has to rely on running effects to generate a nonzero  $\text{Re}[H'_{12}]$ . From Eqs. (7) and (5) we obtain

$$\begin{aligned} \frac{d\text{Re}[H'_{12}]}{dt} &\simeq \left\{ \frac{2c}{\delta_N} (H'_{11} - H'_{22}) \text{Re}[H'_{12}] \right. \\ &\quad \left. - 2a \text{Re}[(Y'^\dagger Y_\ell Y_\ell^\dagger Y')_{12}] \right\}. \end{aligned} \quad (14)$$

Taking into account that  $\text{Re}[H'_{12}(0)] = 0$ , then

$$\text{Re}[H'_{12}(t)] \simeq -\frac{a y_\tau^2}{16\pi^2} \text{Re}[Y'_{31}{}^* Y'_{32}] t, \quad (15)$$

which, in terms of the Yukawa matrix  $Y$ , reads

$$\begin{aligned} \text{Re}[H'_{12}(t)] &\simeq -\frac{a y_\tau^2}{16\pi^2} \{ \text{Re}[Y'_{31}{}^* Y'_{32}] \cos 2\theta \\ &\quad + \sin 2\theta (|Y_{31}|^2 - |Y_{32}|^2)/2 \} t. \end{aligned} \quad (16)$$

The radiatively generated  $\varepsilon_{1,2}$  can be computed from Eqs. (12) and (16).

In the following we will illustrate how the mechanism described above works for a specific example. It is convenient to define the  $3 \times 3$  seesaw operator  $\kappa$  at  $\Lambda_D$ ,  $\kappa = Y Y^T/M$ , where  $Y_{ij} = y_0 y_{ij}$  is a  $3 \times 2$  complex matrix. In order to reconstruct the high energy neutrino sector in terms of the low energy parameters, we choose  $y_{12} = 0$ . The effective neutrino mass matrix  $\mathcal{M}$  is

$$\mathcal{M} = m_3 U \text{diag}(0, \rho e^{i\alpha}, 1) U^T, \quad \rho \equiv m_2/m_3, \quad (17)$$

where  $m_3$  is the mass of the heaviest neutrino and  $\alpha$  is a Majorana phase. In the present case

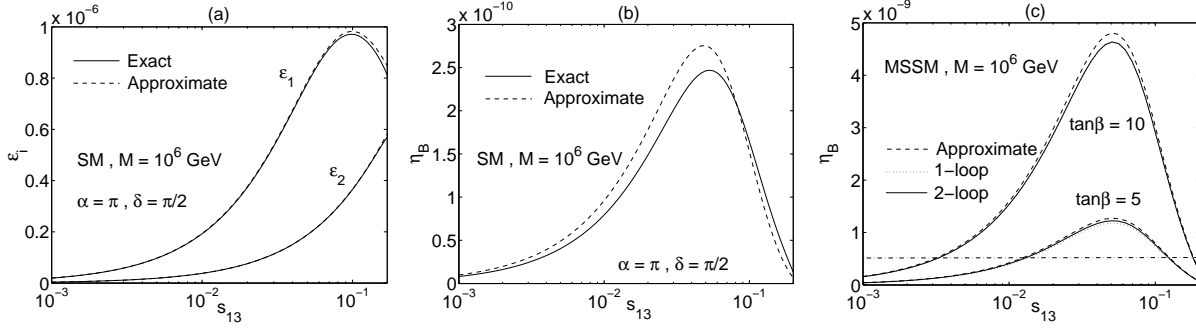


Figure 1.  $CP$  and baryon asymmetries as functions of  $s_{13}$  considering  $\alpha = \pi$ ,  $\delta = \pi/2$ ,  $\Lambda_D = 10^{16}$  GeV and  $M = 10^6$  GeV (see text for more details).

$m_2 = \sqrt{\Delta m_\odot^2}$  and  $m_3 = \sqrt{\Delta m_a^2}$ , where  $\Delta m_\odot^2$  and  $\Delta m_a^2$  are the solar and atmospheric neutrino mass-squared differences measured by neutrino oscillation experiments. When necessary, we will use the best-fit values  $\Delta m_\odot^2 = 8.1 \times 10^{-5} \text{ eV}^2$  and  $\Delta m_a^2 = 2.2 \times 10^{-3} \text{ eV}^2$  [10]. From these, one can determine  $\rho = \sqrt{\Delta m_\odot^2 / \Delta m_a^2}$ .

The matrix  $U$  is the leptonic mixing matrix which can be parametrized, as usual, in terms of three mixing angles  $\theta_{ij}$  and a Dirac-type  $CP$ -violating phase  $\delta$ . The best fit-values for  $\theta_{ij}$  are  $\sin^2 \theta_{12} = 0.3$  and  $\sin^2 \theta_{23} = 0.5$ , where  $\theta_{12}$  and  $\theta_{23}$  denote the atmospheric and solar mixing angles, respectively [10]. For  $\theta_{13}$  we take the  $3\sigma$  bound  $\sin^2 \theta_{13} < 0.047$  [10].

Since the radiative corrections to  $\mathcal{M}$  are negligible in the case where its eigenvalues are hierarchical, one has  $\mathcal{M} \simeq v^2 Y Y^T / M$  which implies  $y_0^2 = M \sqrt{\Delta m_a^2} / v^2$ . In terms of the low-energy neutrino parameters,  $\varepsilon_{1,2}$  take the approximate form [1]

$$\varepsilon_1 \simeq -\frac{3y_\tau^2}{64\pi} \frac{x\sqrt{\rho}(1+\rho)\sin(\alpha/2)}{(1-\rho)(\rho+x^2-\Delta)} [c_{12}\cos(\delta-\alpha/2) + \sqrt{\rho}s_{12}^2x\cos(\alpha/2)], \quad \varepsilon_2 \simeq \frac{\rho+x^2-\Delta}{1+\rho x^2+\Delta} \varepsilon_1, \quad (18)$$

where  $x = \tan \theta_{13} / (\sqrt{\rho} s_{12})$  and

$$\Delta = \frac{1}{2}(1-\rho) \left[ -1 + x^2 + \sqrt{1 + 2x^2 \cos \alpha + x^4} \right].$$

We use the notation  $s_{ij} \equiv \sin \theta_{ij}$  and  $c_{ij} \equiv \cos \theta_{ij}$ . Taking for instance  $\alpha \simeq \pi$  and  $\delta \simeq \pi/2$ , the  $CP$

asymmetry  $\varepsilon_1$  reaches its maximum value for  $x = \sqrt{\rho}$ , as can be readily seen from Eq. (18). This corresponds to  $s_{13} = \rho s_{12} \simeq 0.1$  and

$$\varepsilon_1^{\max} \simeq -\frac{3y_\tau^2 c_{12}}{128\pi} \frac{(1+\rho)}{(1-\rho)} \simeq -10^{-6}. \quad (19)$$

The accuracy of these approximate expressions is shown in Fig. 1.a, where the  $CP$  asymmetries  $\varepsilon_i$  are plotted as functions of  $s_{13}$  taking  $\Lambda_D = 10^{16}$  GeV,  $M = 10^6$  GeV,  $\delta = \pi/2$ ,  $\alpha = \pi$  and assuming  $y_\tau = 0.01$  in the analytical estimates. The solid lines correspond to the full numerical integration of the RGE, while the dashed ones refer to the approximations given in Eq. (18). The comparison of the curves shows that, for values of  $s_{13} \lesssim 0.1$ ,  $\varepsilon_2$  is suppressed with respect to  $\varepsilon_1$ , in accordance with Eq. (18). Also, the true value of  $\varepsilon_1^{\max}$  agrees with Eq. (19).

The out-of-equilibrium Majorana decays are controlled by the parameters  $K_i = \Gamma_i / H(T = M_i)$  where  $H(T) = 1.66 g_*^{1/2} T^2 / M_P$  is the Hubble parameter,  $g_* \simeq 107$  is the number of relativistic degrees of freedom and  $M_P = 1.2 \times 10^{19}$  GeV is the Planck mass. Considering that the entropy remains constant while the universe cools down from  $T \simeq M$  to the recombination epoch, the baryon-to-photon ratio  $\eta_B$  can be estimated using  $\eta_B \simeq -10^{-2} (d_1 \varepsilon_1 + d_2 \varepsilon_2)$ , where  $d_i \leq 1$  are efficiency factors which account for the washout effects. In our case, it can be shown that  $|\varepsilon_2 / K_2| \ll |\varepsilon_1 / K_1|$  which leads to  $\eta_B \simeq -10^{-2} d_1 \varepsilon_1$ .

A simple estimate of the dilution factor

$d_1$  can be obtained from the fit  $d_1 \simeq 0.6 [\ln(K_1/2)]^{-0.6}/K_1$  [11], where  $K_1$  is, in this case, independent of  $M$  and given by

$$K_1 \simeq \frac{44(\rho + x^2 - \Delta)}{\sqrt{1 + 2x^2 \cos \alpha + x^4}}. \quad (20)$$

For  $\alpha \simeq \pi$ ,  $\delta \simeq \pi/2$ , the maximal value of the baryon asymmetry is then attained for  $x \simeq \sqrt{3\rho/(1+\rho)}/3 \simeq 0.23$  and  $s_r \simeq 0.05$ . From Eq. (18) we find  $\varepsilon_1 \simeq -8 \times 10^{-7}$ . Moreover, since Eq. (20) implies  $K_1 \simeq 11$ , then  $d_1 \simeq 4 \times 10^{-2}$  and  $\eta_B^{\max} \simeq 3 \times 10^{-10}$ , which is by a factor of two smaller than the observed baryon asymmetry  $\eta_B = 6.1_{-0.2}^{+0.3} \times 10^{-10}$  [12]. It is worth noticing that this result is weakly dependent (apart from renormalization effects on  $y_\tau^2$ ) on the heavy Majorana neutrino mass scale  $M$ , as can be seen from the approximate expressions given in Eq. (18). In Fig. 1.b we present the computation of  $\eta_B$  as a function of  $s_{13}$ . The dashed line refers to the result using only the decay of  $N_1$  and considering an approximation for the efficiency factor, while the solid line has been obtained solving the full set of Boltzmann equations, considering both the decays of  $N_1$  and  $N_2$ .

In Fig. 1.c  $\eta_B$  is computed for the MSSM case. Regarding the computation of the  $CP$ -asymmetries, a factor of two has to be included in Eq. (12) due to the presence of supersymmetric particles in the decays. Moreover, since  $\varepsilon_{1,2} \propto y_\tau^2$ , we expect an extra enhancement factor of  $(1 + \tan^2 \beta)$  in the MSSM respective to the SM case (see also Ref. [13]). At the end, this leads to an increase of the value of  $\eta_B$ . The dotted and solid lines refer to the calculation where the  $CP$ -asymmetries were computed using the RGE at one and two-loop orders, respectively, for  $\tan \beta = 5, 10$ . The results show that the two-loop running effects can be perfectly neglected. Moreover, the maximum of  $\eta_B$  can be far above the experimental value, which indicates that in the MSSM there is some freedom in the choice of the  $CP$ -violating phases and  $s_{13}$ . As in the SM case, the result based on an approximation equivalent to the one in Eq. (18) (dashed-lines) agree with the exact result.

To conclude, it is worth noting that if the above mechanism is extended to models with

three heavy Majorana neutrinos, then one can obtain values for  $\eta_B$  compatible with the experiment even in the SM case [14].

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